EECS C145B / BioE C165 Spring 2004: Problem Set IV Due April 16 2004

Please read the sections describing the rules for working in groups and the grading policy in the course introduction handout.

Show all code and plots.

Problem 1 (30 points)

The 2D real plane contains an ellipse of intensity 1 centered at the origin. The ellipse has a semi-major axis of length 3 and a semi-minor axis of length 1. The semimajor axis is oriented at 45 degrees with respect to the x-axis.

1. What is the Radon projection value for $\boldsymbol{\theta} = [1/\sqrt{2}\ \ 1/\sqrt{2}\,]^T$ and $s = 0$? Assume the center of rotation is the origin.
2. What is the Radon projection value for $\boldsymbol{\theta} = [1/\sqrt{2}\ \ 1/\sqrt{2}\,]^T$ and $s = 0$ when the center of the ellipse is shifted to $(0.5, -0.5)$?
3. Verify you answers using Matlab. Show code and plots.

Problem 2 (40 points)

1. Using Matlab, find the backprojection of the $\boldsymbol{\theta} = [1/\sqrt{2}\ \ 1/\sqrt{2}\,]^T$ projection of the ellipse described in the first part of the previous question.
2. Take this image and feed it into the program you wrote in Problem Set II. Along which direction(s) are the sinusoids with the greatest amplitudes traveling?
3. Describe the locus of these sinusoidal components in the frequency domain both verbally and mathematically.
4. What did you learn from this problem?

Problem 3 (20 points)

Consider the matrix:

$$\mathbf{F} = \begin{bmatrix} 3 & 4 & 5 \\ 1 & -1 & 6 \end{bmatrix}$$

1. Draw and label the range and nullspace vectors.
2. Try to give an example of a vector that this matrix maps to the zero vector.
3. Try to give an example of a 2D vector that is not in the range of $\mathbf{F}$.

Problem 4 (40 points)

1. Use the Matlab command *phantom* to generate a brain phantom of 256×256 pixels.
2. Using the sampling criteria for the Radon transform and the function *radon*, generate a set of projections and plot the sinogram.
3. Using the function:

   `http://muti.lbl.gov/145b/backproj.m`

   reconstruct the imaged distribution using the backprojection of Fourier-filtered projections algorithm. Use an ideal ramp filter.

Optional problems (not graded)

Optional problem 1

Perform the same analysis for the transpose of $\mathbf{F}$ in Problem 3.

Optional problem 2

The following diagram shows two angular projections of a 4 pixel distribution.

p_{11} 5	p_{12} 11		
μ_1	μ_3	12	p_{21}
μ_2	μ_4	4	p_{22}

1. Solve for the μ using the algebraic reconstruction technique (ART) (see reader).
2. Write a matrix equation relating the projections $\boldsymbol{\lambda}$ to the pixel values $\boldsymbol{\mu}$.

3. Solve for $\boldsymbol{\mu}$ assuming the measured p_{ij} are contaminated with independent Gaussian noise having unit variance.

4. Write out the negative log likelihood function assuming projection bins are realizations of independent Poisson processes.

5. Write out the gradient of this equation (partial derivatives with respect to the μ_n).

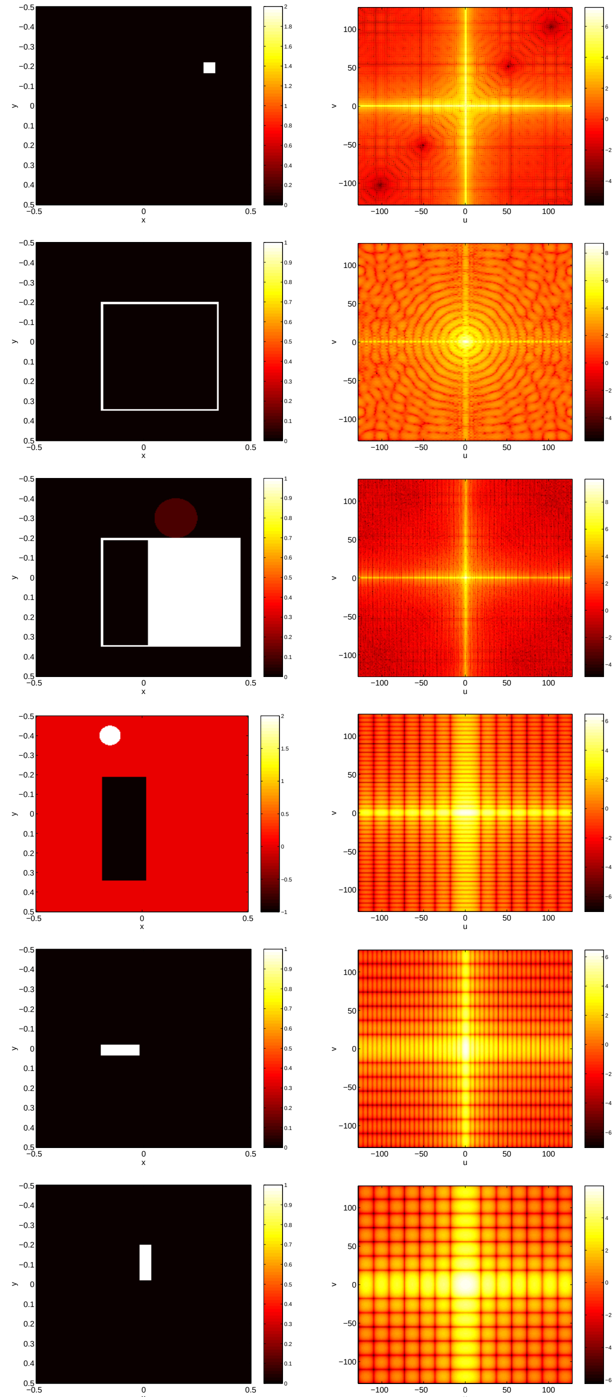
6. Evaluate the direction in which we step in the steepest descent method when we start at $\boldsymbol{\mu}_0 = [4\ 4\ 4\ 4]^T$

Optional problem 3

Prove mathematically that the Radon transform of a 2D Gaussian is a 1D Gaussian.

Optional problem 4

Match the elements in the left column to those in the right that could possibly be the log magnitude of their Fourier transforms. Enter the letter identifying the match on the line to the left of each row.



a

b

c

d

e

f